MATH 611

Homework 4

1. In order to estimate the integral

Consider the following two choices of an importance function.

For n=10000 simulations summarize the results in a table showing the values of the estimates and their standard errors.

1. In order to approximate the value of the simple integral

Consider an antithetic variable approach with . What is the percent reduction in the variance of the estimate that can be achieved using this antithetic approach, compared to simple MC integration?

1. For the transition matrix:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** | **5** |
| **1** | 0.4 | 0.3 | 0.3 | 0 | 0 |
| **2** | 0 | 0.5 | 0 | 0.5 | 0 |
| **3** | 0.5 | 0 | 0.5 | 0 | 0 |
| **4** | 0 | 0.5 | 0 | 0.5 | 0 |
| **5** | 0 | 0.3 | 0 | 0.3 | 0.4 |

Identify the irreducible closed sets of the Markov Chain. Identify the transient and recurrent states.

1. Find the stationary distribution for the Markov Chain on {1,2,3,4}, having transition probability matrix:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **1** | 0.7 | 0 | 0.3 | 0 |
| **2** | 0.6 | 0 | 0.4 | 0 |
| **3** | 0 | 0.5 | 0 | 0.5 |
| **4** | 0 | 0.4 | 0 | 0.6 |

1. For the example ‘Particles in a Box’, presented in class, show that the stationary distribution, , of the Markov Chain is also Poisson and determine the value of the Poisson parameter.